TINET: Triangulated Irregular Networks Evolving in Time

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Abstract

Triangulated Irregular Networks (TIN) are popular structure in Geographic Information Systems (GIS) software, used for representing surfaces via discrete triangular facets. Often times, the 3D vertices are obtained from: (i) a collection of 2D points, corresponding to physical locations; and (ii) a "mass" value corresponding to the third dimension, representing a magnitude of a measurement in each location. The organization of the 2D points for representing TINs is based on Delaunay triangulation, a well known concept from Computational Geometry. In practice, however, measurements in each location are also taken in multiple discrete time instants, thereby inducing a location-bound time series. Combining the values from the locations in discrete time instants, in turn, creates a setting of evolving TINs. This work describes TINET (TINs Evolving in Time) – a methodology for comparing surfaces represented by TINs in different time instants. Such feature is essential for reasoning about aggregated properties of the evolution of a special phenomena, such as examining the similarity/distance between rainfall or CO_2 distribution at different times.

***K*eywords** Triangulated Irregular Networks · Surface distances · Time series

1 Introduction and Motivation

Advances in sensing technologies and the proliferation of location-aware IoT devices have enabled generation of large volumes of spatial data, augmented with semantic contexts such as type of data, measurement frequencies, etc. However, in many applications, the sensed/observed data is also associated with a *location*. For example, measurements of air temperature, CO_2 concentration, rainfall, etc. are done by sensors positioned at fixed locations, either pre-installed or by having users visiting particular locations as part of a participatory sensing effort [1].

Often, one is interested in the spatial distribution of a particular physical phenomena – however, the data can only be measured in discrete locations.

Among the most popular data types in geo-spatial applications to approximate a continuous surface using measurements in discrete locations are TINs (Triangulated Irregular Networks) [2, 3]. TINs are 3D surfaces consisting entirely of triangular faces, and are often implemented as part of functionalities in Geographic Information Systems (GIS) software (e.g., ArcGIS Pro [4]).

The 2D projections of vertices of the triangles typically correspond to locations in which measurements are taken, and the height of each vertex in the TIN corresponds to a value taken at a particular location. One of the main questions when constructing TINs is how to organize the set of 2D points corresponding to the locations into triangles. For that, it turns out that the most popular approach is to generate the Delaunay Triangulation (DT) of the planar points [5].

In this work, we focus on a specific aspect of TIN – namely, we consider settings in which measurements in particular locations are taken regularly over time (i.e., with some sampling frequency). Thus, the collection of measurements at a given location can be perceived as a time series. Much work has been done in terms of investigating different representation methods and distance functions for evaluating similarity of time series [6].

However, such works provide methods that can assess a distance (similarity) between time series at two distinct locations; or detect a location with most similar time series to another/given location [7].

What we aim is to provide means to qualitatively compare TINs at different time instants of their evolution. Such objective could enable quantitative comparison of the differences of distribution of the desired phenomenon (e.g., temperature, precipitation) across an area of interest. Towards that, we propose to augment the existing TIN approaches with methodologies for assessing distance between TINs.

We note that the code and the datasets that we used for testing are publicly available at https://github.com/Prabingiri/DT-TIN-Visualization.

2 Methodology

We now describe in detail the main aspects of the proposed approach. We first recollect some basic terminology and then describe the details of the distance functions between surfaces.

2.1 Triangular Irregular Network (TIN)

As mentioned, TINs are surfaces consisting entirely of non-overlapping triangular facets, possibly in different resolutions, enabling to focus on small details in highly variable input features [8]. In other words, they can be perceived as a special case of Digital Elevation Model (DEM) having the surface of triangular mesh – the base of which is defined by a set T of triangles, constructed from a finite set of S points as their vertices (cf. [3]). Clearly, the boundaries of triangular facets can intersect only in a common edge or vertex.

The (irregularly spaced) sampled points are connected through straight lines to form 2D based triangles and there are multiple choices for generating the set of edges. In practice, however, the Delaunay triangulation (DT) is the most prevalent one [5].

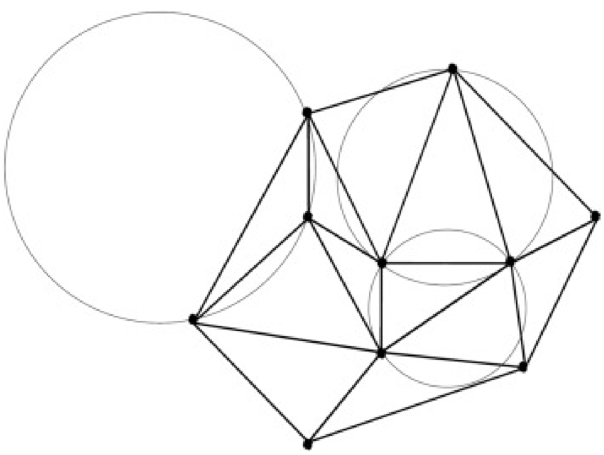

Figure 1: Delaunay Triangulation for constructing TINs

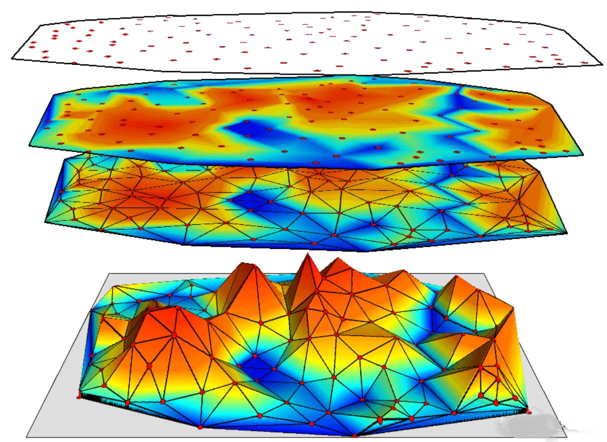

Figure 2: Surfaces of a TIN

DT organizes the triangles in such a manner that the circumscribed circle which passes through vertices of three points, contains no other point from the input S, as shown in Figure 1 (cf. [8, 9]). DTs have numerous properties that have been identified by the researchers in the Computational Geometry community [5]. The most common one is that they are geometric dual of the Voronoi diagram constructed from the set of input points S. However, one of the most interesting properties regarding their use for TINs construction is that they maximize the minimum angle of all the angles of the triangles in the triangulation. The relevant consequence is that this ensures that they avoid the so called, sliver triangles (i.e., ones that will have long/thin shape) – which, in turn, improves the smoothness of the TIN structure.

The intuition behind organizing the 2D locations according to DT, and constructing the TIN from the measurement values in each input location, is illustrated in Figure 2. We used scipy.spatial library for implementing the DT.

2.2 Time series

Commonly, a *time series* is sequence of values $\{v_1, v_2, \ldots, v_n\}$ where each v_i can be perceived as measurement of (the value of) a particular phenomenon at time-instant t_i. In our settings, we consider a collection of multiple time series $\mathcal{T} = \{T_1, T_2, \ldots, T_k\}$ where each T_j is associated with a unique location (lat_j,lon_j)[7].

For a fixed time instant t_i, the collection of values $\{v_{i1}, v_{i2}, \ldots, v_{ik}\}$ corresponds to the measurements obtained at the corresponding locations $\{(lat_1, lon_1), (lat_2, lon_2), \ldots, (lat_k, lon_k)\}$. Thus, one can readily construct a TIN corresponding to the surface of the spatial distribution of the phenomena of interest at the given t_i.

However, we are interested in providing some kind of a global measure of distances between two distinct TINs (i.e., the distribution of the phenomenon at two distinct time instants). An illustration is provided in Figure 3 which shows the distribution of rainfalls in a given spatial range. The data corresponds to the precipitation measurements of different weather stations across the globe (https://climatedataguide.ucar.edu/climate-data/gpcc-global-precipitation-climatology-centre).

2.3 Evolving TINs and Surface Distances

We next describe how to evaluate the distance between TINs in different time-instants.

2.3.1 Hausdorff distance

Haudsorff distance is a min-max distance measure which can be used to assess the similarity between two surfaces based on the set of locations. It is widely used as a measure of the degree of resemblance between two objects [10].

For two given finite sets of points A and B, such that $A = a_1, a_2 \ldots . a_n$ and $B = b_1, b_2 \ldots . b_n$, Hausdorff's distance is defined as

$$H(A, B) = \max(h(A, B), h(B, A)), \tag{1}$$

where

$$h(A, B) = \max_{a \in A}(\min_{b \in B}(d(a, b))) \tag{2}$$

and

$$h(B, A) = \max_{b \in B}(\min_{a \in A}(d(b, a))) \tag{3}$$

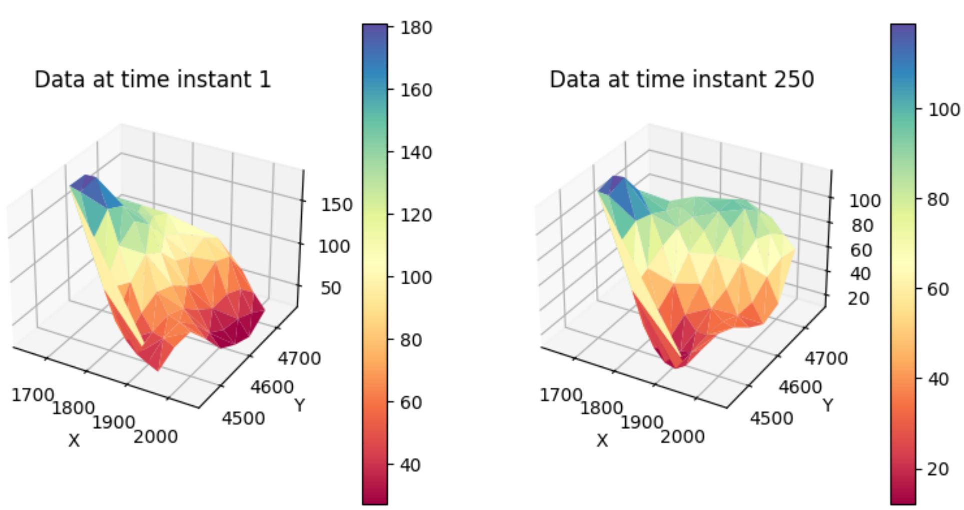

Figure 3: 3D plot of TINs at different time instants

We note that since TINs have only triangular faces, at least one of the points from one of the surfaces is located in a vertex or along an edge. We used the *directed_hausdorff* from scipy.spatial library in our implementation.

2.3.2 Volume Based Distance

The second distance function that we consider is based on comparing the volumes of the TINs obtained from the different time instants, as the base triangles (i.e., the locations of the measurements) are fixed. Volume similarity measure has been widely used as one of the techniques to measure the similarity between segments [11], and we developed the version from [12].

Each triangle of the TINs are considered as truncated triangular prism having different heights at a particular time instance, corresponding to the measure value at the base point. Figure 4 illustrates a truncated prism for a single base triangle and portions of two different TINs.

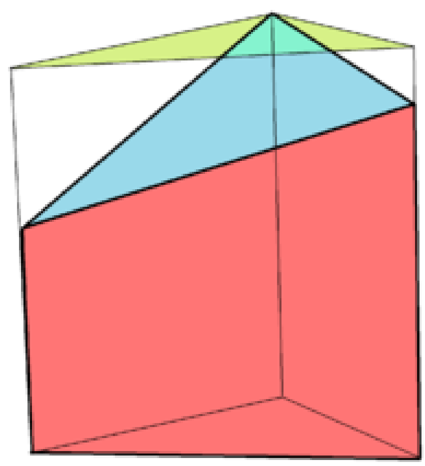

Figure 4: Truncated triangular prism [12]

For a given prism with a base consisting of a triangle with vertices a, b, c, and a height C, the volume is:

$$V = \frac{1}{2}|(\vec{ab} \times \vec{ac})| \cdot C \tag{4}$$

where C corresponds to the average value of the measurements recorded in the three weather stations (i.e., $C = (\textit{height}(a) + \textit{height}(b) + \textit{height}(c))/3$), the locations of which constitute the vertices of the triangle.

3 Conclusion and future work

We presented TINET, a methodology for comparing the evolution of TINs over time. Such collections of spatial surfaces are obtained from time series, each of which is bound to a location at which a particular phenomenon is being periodically sensed.

Part of the future work incorporates extending the functionality of TINET to cater to scenarios involving mobile users – i.e., TINs based at different locations for different time instants. Such scenarios are common in participatory sensing, where mobile users are asked to provide measurements of a particular phenomenon. In addition to the measurement of a phenomenon at different time instants being bound to different locations, these settings also entail non-uniformity of the time series provided by different users.

Lastly, we plan to investigate novel data structures and algorithms, specifically targeting efficient processing of specific queries such as *Detect the counties which have most similar distribution of the precipitation, to DuPage county, within afternoon hours.*

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